From free algebras to proof bounds

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Abstract

(This is joint work with Nick Bezhanishvili)[1],[2]. In the first part of our contribution, we review and compare existing constructions of finitely generated free algebras in modal logic focusing on step-by-step methods. We discuss the notions of step algebras and step frames arising from recent investigations [3], [2], as well as the role played by finite duality. A step frame is a two-sorted structure which admits interpretations of modal formulae without nested modal operators.

In the second part of the contribution, we exploit the potential of step frames for investigating proof-theoretic aspects. This includes developing a method which detects when a specific rule-based calculus Ax axiomatizing a given logic L has the so-called *bounded proof property*. This property is a kind of an analytic subformula property limiting the proof search space. We prove that every finite conservative step frame for Ax is a p-morphic image of a finite Kripke frame for L iff Ax has the bounded proof property and L has the finite model property. This result, combined with a 'step version' of the classical *correspondence theory*, turns out to be quite powerful in applications. For simple logics such as $\mathbf{K}, \mathbf{K4}, \mathbf{S4}$, etc, establishing basic matatheoretical properties becomes a completely automatic task (the related proof obbligations can be instantaneously discharged by current first-order provers). For more complicated logics, some ingenuity is still needed, however we were able to successfully apply our uniform method to Avron's cut-free system for **GL** and to Goré's cut-free system for **S4.3**.

References

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