Fuzzy Non-monotonic Logic Using Two Membership Functions Known and Unknown

Venkata Subba Reddy Poli

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***Abstract*—John McCarthy proposed non-monotonic reasoning for incomplete information in which reasoning is changed if knowledge is added to the system. Non-monotonic reasoning. Nonmonotonic Problems are undecided. An undecided problem has no solution. A method needed to solve undecided AI problems. In this paper, fuzzy modeling for non-monotonic logic is studied as method for non-monotonic reasoning. The Fuzzy non-monotonic reasoning is studied with a twofold fuzzy logic. Fuzzy truth maintenance system (FTMS) is studied for fuzzy non-monotonic reasoning. Fuzzy logic programming is given for non-monotonic reasoning some examples are discussed for fuzzy non-monotonic reasoning.**

Keywords*— *fuzzy Sets, twofold fuzzy sets, non-monotonic reasoning, fuzzy non-monotonic reasoning, incomplete knowledge, FTMS, fuzzy logic programming

I. INTRODUCTION

AI has to deal with incomplete knowledge. If knowledge base is incomplete then the inference is also incomplete. If knowledge is added than the inference is changes in non-monotonic reasoning. Though the non-moronic rezoning is used for some incomplete AI problems, still the reasoning is in complete. Some knowledge is not sufficient for reasoning. If added some knowledge, it sufficient for reasoning in non-monotonic [4].

In non-monotonic reasoning, if additional information is added, the reasoning will be changed or jumping conclusion [4].

X is bird Λ x has wings Λ x is known to fly □x can fly

Suppose

x is bird Λ x has wings Λ x is unknown to fly □x can fly

or

x is bird Λ x has wings Λ x is unknown to fly □x can't fly

Ozzie is bird Λ Ozzie has wings Λ Ozzie x is known to fly □ Ozzie can fly

Ozzie is bird Λ Ozzie has wings Λ Ozzie x is unknown to fly □ Ozzie can't fly

Ozzie is bird Λ Ozzie has wings Λ Ozzie is unknown to fly □ Ozzie can fly

∀xP(x)Λ∀xQ(x)Λ∀xR(x)□∀xS(x)

In monotonic

∀x bird(x)Λ∀x Wings(x)Λ∀x known-to-fly(x)□∀x fly(x)

In non-monotonic

∀x bird(x)Λ∀x Wings(x)Λ∀x unknown-to-fly(x)□∀x can-fly(x) or ∀x can't-fly(x) or

The conclusion will be changed if added some knowledge in non-monotonic logic.

These problems fall under undecided. The undecided problems have no solution. But still the undecided problem has solution with fuzzy logic.

Consider the rule

x is A and x is B then x is D.

If some knowledge is added, to rule the conclusion will be changed.

"x is A and x is B and x is C then x is E.

There are many theories to deal with incomplete information like Probability, Dempster- Shaffer theory, Possibility, Plausibility etc. Zadeh [11] fuzzy logic is based on belief rather than probable (likelihood). The fuzzy logic made imprecise information in to precise.

Zadeh fuzzy logic is defined with single membership function.

Fuzzy logic with two membership functions will give more information

Two fold fuzzy logic P=(A, B) for the proposition of the type "x is P". A is supporting the knowledge and B is against the knowledge.

P may be considered as

P={belief, disbelief}, {True, false}, {Known, unknown}, {belief, disbelief} etc.

x is bird Λ x has □ x can fly

x is bird Λ x has wings □ x can fly

$\mu_P(x)$ Λ $\mu_Q(x)$ Λ $\mu_R(x)$ □ $\mu_S(x)$

where P,Q and S are twofold fuzzy set known, known}.

The conflict of the incomplete information may be defend by fuzzy certainty factor(FCF)

FCF P = (A-B)

FCF P = (unknown- known)

Where known and unknown are the fuzzy membership functions.

The fuzzy non-monotonic reasoning will bring uncertain knowledge in to certain knowledge.

$\mu_P(x)^{(unknown, known)}$ Λ $\mu_Q(x)^{(unknown-,known)}$ □ $\mu_S(x)$

where S is quasi fuzzy set i.e. S=[0,1].

$\mu_{bird}(x)^{(unknown, known)}$ Λ $\mu_{wings}(x)^{(unknown-,known)}$ □ $\mu_{fly}(x)$

II. FUZZY LOGIC

The possibility set may be defined for the proposition of the type “x is P” as

$\pi_P(x)$□[0,1]
$\pi_P(x)=\max\{ \mu_P(x_i) \}, x \epsilon X$

$\mu_P(x)= \mu_P(x_1)/x_1+ \mu_P(x_2)/x_2+\ldots+ \mu_P(x_n)/x_n$
$\mu_{bird}(x)= \mu_{bird}(x_1)/x_1+ \mu_{bird}(x_2)/x_2+\ldots+ \mu_{bird}(x_n)/x_n$

$\mu_{bird}(x)=$ 0.0/penguin +0.2/Ozzie+ . 0.6/parrot+ 0.7/waterfowl + 0.9/eagle

Let P and Q be the fuzzy sets, and the operations on fuzzy sets are given below [10]

$P \vee Q=\max(\mu_P(x) , \mu_Q(x)\}$ Disjunction
$P \wedge Q=\min(\mu_P(x) , \mu_Q(x)\}$ Conjunction
$P'=1- \mu_P(x)$ Negation
$PxQ=\min \{ \mu_P(x) , \mu_Q(x)\}$ Relation
$P \text{ o } Q==\min\{\mu_P(x), \mu_Q(x, x)\}$ Composition

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as
$\mu_{very}(x) =\mu_P(x)^2$ Concentration
$\mu_{more\ or\ less}(x) = \mu_P(x)^{0.5}$ Diffusion
The fuzzy rules are of the form “if <Precedent Part> then <Consequent Part>”

The Zadeh [10] fuzzy condition inference s given by

if x is P_1 and P_2 …. X is P_n then Q =
$\min 1, (1-\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x)) +\mu_Q(x)\}$ (2.1)

The Mamdani [5] fuzzy condition inference s given by
if x is P_1 and P_2 …. X is P_n then Q =
$\min \{\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x) , \mu_Q(x)\}$ (2.2)

The fuzzy condition inference “Consequent Part” may be drown drawn from “Precedent Part” Reddy[7]
if x is P_1 and P_2 …x is P_n then Q= x is P_1 and P_2 …. x is P_n
using Mamdani fuzzy conditional inference
if x is P_1 and P_2 …x is P_n then x is P_1 and P_2 …. x is P_n
$= \min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$ (2.3)
.
For instance, x is bird Λ x has wings □ x can fly
x can fly= x is bird Λ x has wings

Quasi-fuzzy set

A quasi-fuzzy set is defined for the proposition “ x is P” as

$\mu_P(x)$□(0, 1)
$\mu_{fly}(x)^{(can,\ can't)}$□(0,1)

III. THE TWO FOLD FUZZY LOGIC

Zadeh Proposed fuzzy set with single membership function. The two fold fuzzy set will give more evidence than single membership function.

The fuzzy non-monotonic set may defined with two fold membership function using unknown and known

Definition: Given some Universe of discourse X, the proposition “ x is P” is defined as its two fold fuzzy membership function as

$\mu_P(x) = \{\mu_P^{unknown}(x), \mu_P^{known}(x)\}$
or
$P = \{\mu_P^{unknown}(x), \mu_P^{known}(x)\}$
Where P is Generalized fuzzy set and x Є X ,

$0 <= \mu_P^{unknown}(x) <=1$ and, $0 <= \mu_P^{known}(x) <=1$
$P = \{ \mu_P^{unknown}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n,$
$\mu_P^{known}(x_1)/x_1 + \ldots + \mu_P^{unknown}(x_n)/x_n$, x_i Є X, “+” is union

For example ‘ x will fly” , fly may be given as
Suppose P and Q is fuzzy non-monotonic sets. The operations on fuzzy sets are given below for two fold fuzzy sets.

Negation
$P'= \{1- \mu_P^{unknown}(x), 1- \mu_P^{known}(x) \}/x$

Disjunction
$P \vee Q=\{ \max(\mu_P^{known}(x) , \mu_P^{known}(y)), \max(\mu_Q^{unknown}(x) , \mu_Q^{unknown}(y))\}(x,y)$

Conjunction
$P \wedge Q=\{ \min(\mu_P^{known}(x) ,\mu_P^{known}(y)), \min(\mu_Q^{unknown}(x) , \mu_Q^{unknown}(y)) \}/(x,y)$

Implication
Zadeh [10] fuzzy conditional inference
P□Q= $\{\min(1, 1- \mu_P^{known}(x) + \mu_Q^{known}(y) , \min (1, 1- \mu_P^{known}(x) + \mu_Q^{known}(y)\}(x,y)$

Mamdani [5] fuzzy conditional inference
P□Q= $\{\min(\mu_P^{unknown}(x) , \mu_Q^{unknown}(y) , \min (\mu_P^{known}(x) , \mu_Q^{known}(y)\}(x,y)$

Reddy [7] fuzzy conditional inference
P□Q= $\{\min (\mu_P^{unknown}(x) , \mu_P^{known}(y)\}(x,x)$

Composition
$P \text{ o } R = \{\min_x (\mu_P^{unknown}(x), \mu_P^{unknown}(x)), \min_x(\mu_R^{known}(x), \mu_R^{known}(x))\}/y$

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as

Concentration
“x is very P
$\mu_{very\ P}(x) = \{ \mu_P^{unknown}(x)^2, \mu_P^{known}(x)\mu_P(x)^2 \}$

Diffusion
“x is more or less P”

$\mu_{more\ or\ less\ P}(x) = (\mu_P^{unknown}(x)^{1/2}, \mu_P^{known}(x)\mu_P(x)^{0.5}$

For instance, consider logical operations on P and Q

P = { $0.8/x_1 + 0.9/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.7/x_4 + 0.6/x_5$}

Q = { $0.9/x_1 + 0.7/x_2 + 0.8/x_3 + 0..5/x_4 + 0.6/x_5$,
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.5/x_4 + 0.7/x_5$}

P V Q = { $0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.6/x_4 + 0.6/x_5$,
$0.4/x_1 + 0.5/x_2 + 0.6/x_3 + 0.7/x_4 + 0.7/x_5$}

P Λ Q = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5$}

P′ = not P= { $0.2/x_1 + 0.1/x_2 + 0.3/x_3 + 0.4/x_4 + 0.5/x_5$,
$0.6/x_1 + 0.7/x_2 + 0.6/x_3 + 0.3/x_4 + 0.4/x_5$}

P□ Q = { $1/x_1 + 0.8/x_2 + /x_3 + 0.9/x_4 + 1/x_5$,
$1/x_1 + 1/x_2 + 1/x_3 + 0.8/x_4 + 1/x_5$}
P o Q = { $0.8/x_1 + 0.7/x_2 + 0.7/x_3 + 0.5/x_4 + 0.5/x_5$,
$0.4/x_1 + 0.3/x_2 + 0.4/x_3 + 0.5/x_4 + 0.6/x_5$}

$\mu_{very\ P}(x) = \{ \mu_P^{unknown}(x)^2, \mu_P^{known}(x)\mu_P(x)^2 \}$
= { $0.64/x_1 + 0.81/x_2 + 0.49/x_3 + 0.36/x_4 + 0.25/x_5$,
$0.16/x_1 + 0.09/x_2 + 0.16/x_3 + 0.49/x_4 + 0.36/x_5$

$\mu_{more\ or\ less\ P}(x) = (\mu_{Unknown}(x)^{1/2}, \mu_{Known}(x)\mu_P(x)^{1/2} \}$
={ $0.89/x_1 + 0.95/x_2 + 0.84/x_3 + 0.77/x_4 + 0.70/x_5$,
$0.63/x_1 + 0.55/x_2 + 0.63/x_3 + 0.81/x_4 + 0.77/x_5$}

The fuzzy certainty factor (FCF) is defined by fuzziness instead of probability for the fuzzy preposition of the type " x is A"
CF[x, A]=MB[x, A]-MD[x,A],
The FCF is the difference between "unknown" and "known" and will eliminate conflict between "unknown" and "known" and, made as single membership function

$\mu_A^{FCF}(x) = \mu_A^{unknown}(x) - \mu_A^{known}(x)$

Quasi-fuzzy set

A quasi-fuzzy set is defined for the proposition " x is P" as

$\mu_P(x)$□(0, 1)
$\mu_A^{unknown}(x)=1$
$\mu_A^{FCF}(x) = 1 - \mu_A^{known}(x)$

where α∈[0,1] and α-cut is decision factor.

$\mu_{fly}^{FCF}(x) = \{1 - \mu_{fly}^{known}(x)\}$
= {1.0/penguin +1.0/Ozzie+ 1.0/parrot+ 1.0/waterfowl + 1.0/eagle - 0.9/penguin +0.7/Ozzie+ . 0.3/parrot+ 0.15/waterfowl + 0.1/eagle }

= { 0.0/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }
For instance " x can fly" for α>=0.5
Is given as

{ 0.0/penguin+0.0/Ozzie+1/parrot+0.65 /waterfowl + 1/eagle }

The inference is given by
Penguin and Ozzie can't fly
Parrot, waterfowl and eagle can fly

IV. Fuzzy NON-monotonic Logic

Since formation of the fuzzy non-monotonic logic is simply two fold fuzzy logic , the non-monotonic proposition may be represented with two fold fuzzy set
$\mu_P(x) = \{\mu_P^{unknown}(x), \mu_P^{nunknown}(x) \}$
where $\mu_P^{unknown}(x)=1$

For instance,

$\mu_{bird}(x) = \{\mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x) \}$

$\mu_{bird}^{FCF}(x) = \{1 - \mu_{bird}^{known}(x)\}$
= {1.0/penguin +1.0/Ozzie+ 1.0/parrot+ 1.0/waterfowl + 1.0/eagle - 0.9/penguin +0.7/Ozzie+ . 0.3/parrot+ 0.15/waterfowl + 0.1/eagle }

= { 0.0/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

$\mu_{bird}(x) = \{ \mu_{bird}^{unknown}(x), \mu_{bird}^{known}(x)\}$
$\mu_{bird}^{FCF}(x) = \{ 1- \mu_{bird}^{known}(x)\}$

$\mu_{bird}^{FCF}(x)$ = {1.0/penguin +1.0/Ozzie+ . 0.8/parrot+ 0.85/waterfowl + 0.9/eagle – 1.0/penguin +0.1/Ozzie+ . 0.1/parrot+ 0.5/waterfowl + 0.0/eagle }
= {0.0/penguin +0.1/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle }

$\mu_{wings}(x) = \{\mu_{wings}^{unknown}(x), \mu_{wings}^{unnown}(x)\}$
$\mu_{wings}(x) = \{1- \mu_{wings}^{known}(x)\}$

$\mu_{wings}(x)$= {1.0/penguin +1.0/Ozzie+1.0/parrot+ 1.0/waterfowl + 1.0/eagle – 1.0/penguin +0.9./Ozzie+ . 0.2/parrot+ 0.1/waterfowl + 1.0/eagle }
where $\mu_{wings}^{unknown}(x)$ is quasi fuzzy set.
The FCF of wings is given by

$\mu_{wings}^{FCF}(x)$= {0.0/penguin +0.1/Ozzie+ 0.8/parrot+ 0.9/waterfowl + 1.0/eagle }

x can fly may be given as using Reddy fuzzy conditional inference "consequent part "may be derived from "precedent part".

Using (2.3), the fuzzy conditional inference is given by
$\mu_P(x) \Lambda \mu_Q(x)$ □ $\mu_S(x)$
$\mu_S(x) = \mu_P(x) \Lambda \mu_Q(x)$

x is bird Λ x has wings x can fly
x can fly =min { x is bird , x has wings }

$\mu_{fly}^{FCF}(x)$= 0.0/penguin +0.0/Ozzie+ . 0.7/parrot+ 0.8/waterfowl + 0.9/eagle

The inference of " x can fly" for $\alpha>=0.5$ is given by
= 1/parrot+ 1/waterfowl + 1/eagle

The inference of " x can fly" for $\alpha <0.5$ is given by
= 0/penguin +0/Ozzie

The parrot, waterfowl and eagle can fly.
The penguin and Ozzie can't fly
Here fuzzy logic made imprecise information to precise information's. Some birds can fly and some birds can't fly.
The fuzzy decision sets or quasi fyzzy set is defined by

$$R= \mu_A^R(x) = 1 \quad \mu_A^{FCF}(x)\le\alpha,$$
$$0 \quad \mu_A^{FCF}(x)>\alpha$$

Whre R is quasi fuzzy set

For instance, The parrot, waterfowl and eagle can fly and, penguin and Ozzie are can't fly.

V. FUZZY TRUTH MAINTENANCE SYSTEM

Doyel [3] studied truth maintenance system TMS] for non-monotonic reasoning
The fuzzy truth maintenance system (FTMS) for fuzzy non-monotonic reasoning using fuzzy conditional inference as

if x is x is P_1 and P_2 …. And x is P_n then Q
$=\min(\mu_{P1}(x), \mu_{P2}(x), \ldots, \mu_{Pn}(x))$
FTMS is having There is list of justification and conditions.

List L(IN-node, OUT-node)
IN node is unknown fuzzy information.
OUT node is known fuzzy information
Condition (consequent)

if x is bird and x has wings then x can fly

L1 bird(unknown, known)
L2 wings(unknown, known)

Condition fly

The FTMS gives usinf FCF
L1 bird(1, 0.6)
L2 wongs(1, 0.7)
L1 bird(1-0.6)
L2 wings(1-0.7)
L1 bird(0.4)
L2 wings(0.3)

L1 Λ L2 = 0.3

Condition fly=0.3
x can fly $\alpha<=0.4$
x can't fly >0.4

VI. FUZZY MODULATIONS AND LOGIC PROGRAMMING

The fuzzy reasoning system(FRS) is complex reasoning system for incomplete AI problem solving. The fuzzy predicate logic (FPL) is modulating transform fuzzy facts and rules in to meta form(semantic form). These fuzzy facts and rules are modulated to represent the knowledge available to the incomplete problem.

The fuzzy modulations for Knowledge representation are type of modules for fuzzy propositions "x is A".
"x is A" is may be represented as
[A]R(x),
where A is twofold fuzzy set {unknown, known}, R is relation and x is individual in the Unversed of discourse X.
For instance
"x is bird" is modulated as
[bird]is(x)

The FPL is e combined with logical operators.
Let A and B be two fold fuzzy sets.
x is ¬A
[¬A]R(x)
x is A or x is B
[A V B]R(x)
x is A and x is B
[A Λ B]R(x)
if x is A then x is B
[A → B]R(x)

x is bird
[bird]is(x)
if x is bird then x can fly
if [bird]is(x) then [fly]is(x)
or
[bird][fly]is(x)
if x is bird and x has wings then x can fly
"x is bird Λ x has wings Λ x can fly"

if [bird]is(x)Λ[wings]has[x] then [fly]can(x)

if [bird]is(x)Λ[wings]has[x] then [fly]can(x)
[fly]can(x)= { [bird]is(x)Λ[wings]has[x]
The Logic Programming may be written in SWI-Prolog as

```
fuzzy(Ozzie, A,B, M) :- A<B, M is A.

fuzzy(Ozzie, A,B, M) :- A>=B, M is B.

fuzzy(C,M,F):-C<M,F is C.

fuzzy(C,M,F):-C >=M F is M.

fuzzy(X, A,B,C,F) :- fuzzy(X, A,B,M), fuzzy(C,M,F).
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?-run(X,0.3,0.4,0.4.5,F).
F=0.3
If F <=0.4, Ozzie can fly
?-run(Ozzie,0.6,0.5,0.4.5,F).
F=0.45
If F >-4, Ozzie can't fly

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