A Study of Interaction Diagrams of Irregularly Shaped Reinforced Concrete Column with Hollow Cross Section Using the Closed Polygon Method.

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A STUDY OF INTERACTION DIAGRAMS OF IRREGULARLY SHAPED REINFORCED CONCRETE COLUMN WITH HOLLOW CROSS SECTION USING THE CLOSED POLYGON METHOD.

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1. ABSTRACT

Typically, reinforced concrete (RC) columns use rectangular or circular geometric configurations. In certain unique scenarios, a shape needs to appear irregular or asymmetrical, as well as have many holes in its cross section. The need for this special design arose from the requirements imposed by the need for plumbing holes and holes for cable routes. Analysis of irregular cross-sections of RC columns requires a more comprehensive examination of interaction diagrams. This research aims to develop interaction diagrams and computer programs specifically designed for the analysis of RC columns which are characterized by irregular cross-sections containing several holes. The method for analyzing the cross-sectional configuration of irregular RC columns through the use of the closed polygon method requires assigning numerical labels to the vertices in a counterclockwise manner for the outer boundary (exterior-boundary), and clockwise for the use of the inner boundary (Interior-boundary). This approach differs from various techniques used by previous researchers, in that they combine interior boundaries with exterior boundaries to form integrated polygon boundaries. The analysis results and computer program output were evaluated and validated using PCACOL software output, which revealed near accurate findings. With the NMSE (Normalized Mean Square Error) performance index test to measure how much alignment and accuracy there is, for cross-sectional rotation 0 degree the results are 0.002147 for axial force and 0.0007836 for bending moment. And than for cross-sectional rotation 25 degree the results are 0.000234253 for axial force and 0.000137014 for bending moment. So it has high accuracy

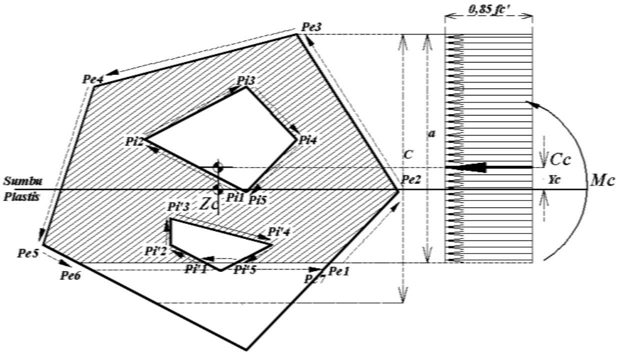
Keywords: *Interaction diagram, irregular shape, closed polygon method*

2. INTRODUCTION

Reinforced concrete column interaction diagram is a graph that describes the interaction between axial force capacity and bending moment that occurs in the column. This diagram functions as a visual guide in providing information regarding its ability to withstand axial loads and bending moments that occur in pairs. For reasons of aesthetic function and beauty, these columns are generally made in the shape of a square or circle. However, due to demands from; mechanical & electrical, and plumbing, the column also functions for hoist-cable-try, and hoist-plumbing, so a column with a hole in the middle is needed.

How to analyze the strength of irregular shaped columns generally uses the help of computer application programs because manual calculations are more complicated and complex. Helgason (2010), explains that a symmetrical, rectangular column that experiences a biaxial bending moment in both directions of the major and minor axes simultaneously, bending will not occur on the major axis. Due to the changing direction of the bending angle, the plastic axis of the column cross-section divides the two parts which are no longer symmetrical. The plastic axis in the column cross-section becomes irregular. In calculating the area of compression to analyze the cross-section of a column that is irregular and has holes in its cross-section, there are several ways and methods; Fattah et al (2017), Ghoneim & Mahmoud (2008) have modeled the column cross-section divided into several meshes or networks of smaller discrete elements. Each mesh has a small area and the distance of the center of gravity to the plastic axis of the cross section is easier to obtain. Hulse & Mosley (1986) have calculated the area of the compression block and the center of gravity of the hexagon column section which is categorized as a non-rectangular section. The direction of numbering the vertices is determined clockwise. The cross-sectional area is calculated using the closed polygon method. The center of gravity of the cross-section is calculated by dividing the first-moment-of-area about each axis divided by the cross-sectional area. Greulich (1995), Ranjbaran (1992), Kwan & Liauw (1985), Marin (1983) according to them how to calculate the cross-sectional area of an irregular column that has a hole in the middle or Interior-boundary, and the column experiences a two-way bending moment or biaxial bending , calculated using the closed polygon method. The direction of numbering the vertices is counterclockwise. The interior-boundary is assembled into one unit with the exterior-boundary. The center of gravity of the cross-sectional area is calculated by dividing the first-moment-of-area about each main axis divided by the cross-sectional area of that area.

Based on these seven methods, we present several elements that are different from the seven methods mentioned previously. Method for calculating the area of the exterior-boundary concrete compression area and the distance of the center of gravity to the plastic axis of



various cross-sections of any shape represented as a closed polygon with the direction of numbering the vertices counterclockwise. So that the area area calculation results are positive, the differentiating factor with the seven previous methods is the direction of interior-boundary node numbering. That is, it must be arranged clockwise so that the calculated cross-sectional area is negative. The next differentiating factor is that each interior-boundary stands alone and is not assembled into one unit with other exterior-boundaries. To get a smooth interaction diagram graph it takes at least around 26 neutral axis iterations. This is work that is done repeatedly and has the potential for miscalculations, so a consistent tool is needed for calculating, namely an application program on a computer. Meanwhile, the control method results from manual calculations and computer programs were compared with the output from PCA-COL.

3. METHODS

After knowing the variable data for concrete quality f'c, steel quality fy, modulus of elasticity of steel Es, max strain of concrete εcu. Then follows the boundary-section data which consists of the number of boundaries, followed by data on the number of nodes for each boundary along with the coordinates Xc and Yc.

Section Properties For Concrete :

f'c	28	Mpa				
Ext Boundary	Pe1	Pe2	Pe3	Pe4	Pe5	Pe6
x-coordinate	28,0303	178,0303	78,0303	-121,9697	-171,9697	28,0303
y-coordinate	-152,2727	-2,2727	147,7273	97,7273	-52,2727	-152,2727
Int Boundary	Pi1	Pi2	Pi3	Pi4	Pi5	
x-coordinate	28,0303	-71,9697	28,0303	78,0303	28,0303	
y-coordinate	-2,2727	47,7273	97,7273	47,7273	-2,2727	
Int Boundary	Pi1'	Pi2'	Pi3'	Pi4'	Pi5'	
x-coordinate	3,0303	-46,9697	-46,9697	53,0303	3,0303	
y-coordinate	-77,2727	-52,2727	-27,2727	-52,2727	-77,2727	

Properties of Reinforcement :

Id	fy	Es	ds	as
As(1)	400	200000	34,9723	529,7633
As(2)	400	200000	72,8912	529,7633
As(3)	400	200000	142,9321	529,7633
As(4)	400	200000	177,4709	201,0619
As(5)	400	200000	255,7834	529,7633

The vertex numbering direction for the exterior-boundary is counterclockwise, while for the interior-boundary it is clockwise. Pe is the exterior node, while Pi and Pi' represent the interior node. As in Figure 2. By changing the location of the neutral axis C, it will cause changes in the size of the concrete compression area and different stresses in the reinforcing steel, resulting in different moment capacities and axial forces. For one step of the neutral axis, the new polygon-boundary coordinates will be obtained which are above the line at height a = C.β1. The factor β1 is taken as follows:

(a) For concrete strength, ***f'c*** up to 28 MPa, ***β1***= 0.85

(b) For concrete 28 MPa < ***f'c*** ≤ 56 MPa,
β1=0.85 – 0.05 (***f'c*** - 28Mpa)/7Mpa......................... (1)

(c) For concrete greater than 58 Mpa, ***β1***=0.65

So we rearrange the concrete area at height a as new boundary data. Next, calculate the area and center of gravity regarding the plastic center Zc.

Figure 1. Boundary node numbering and certain C values along with Cc & Mc

Next, calculate the area and center of gravity of each polygon-boundary:

$$Ac = \sum_{i=1}^{n} Api \qquad (2)$$

$$Api = 1/2\left((x_i . y_{i+1}) - (x_{i+1} . y_i)\right) \qquad (3)$$

$$xc = \frac{1}{Ac}\sum_{i=1}^{n}\frac{1}{3}Api(\, x_i + x_{i+1}) \qquad (4)$$

$$yc = \frac{1}{Ac}\sum_{i=1}^{n}\frac{1}{3}Api(y_i + y_{i+1}) \qquad (5)$$

Ap(i..n) is the area covered by one step from node *i* to the next node ***i+1***, ***Ac*** is the area of one cross-sectional boundary of a closed polygon, ***x(i...n)*** is the x coordinate of each polygon vertex, ***y(i...n)*** is the y coordinate of each polygon vertex, ***xc*** is the ***x*** coordinate of the centroid of one boundary ***Ac, yc*** is the y coordinate of the centroid of one boundary ***Ac***, ***n*** is the number of vertices of each polygon-boundary. If there is more than one boundary polygon then the total column cross-sectional area and center of gravity will be:

$$Act = \sum_{i=1}^{nb} Ac \qquad (6)$$

$$Xc = \frac{1}{Act}\sum_{i=1}^{nb} Ac.xc \qquad (7)$$

$$Yc = \frac{1}{Act}\sum_{i=1}^{nb} Ac.yc \qquad (8)$$

Act is the total area of the polygon-boundary, ***Ac*** is the area of each i-polygon-boundary, ***Xc*** is the ***X*** coordinate of the total center of gravity of the Act section, and ***Yc*** is the ***Y*** coordinate of the center of total gravity of the Act section, ***xc(i…n)*** is the ***x*** coordinate of the Center of mass of each i-polygon-boundary, ***yc(i...n)*** is the ***y*** coordinate of the center of gravity of each i-polygon-boundary, ***nb*** Number of closed n-polygon-boundaries. With the catch point of the shaded cross section being at the height of ***Yc***. And ***Ac*** is the cross-sectional area of the concrete pressed block. The magnitude of the compressive force due to the pressed concrete section at height a is :

$$Cc = 0.85.\, f'c.\, Act \qquad (9)$$

Cc is the capacity of the compressive force due to the concrete cross-section at height a, while the moment of resistance to the center of the column cross-section caused by the concrete pressing block at height a towards the plastic center of the cross-section is:

$$Mc = 0.85\, .\, f'c . Act\, . Yc \qquad (10)$$

Mc is the Moment of Capacity caused by the concrete section at height ***a***, ***Yc*** The height of the center of gravity of the boundary section at height a relative to the plastic center (0,0)

The next data is the number of reinforcement points followed by the ***Xs*** and ***Ys*** coordinates of each reinforcement along with the area ***As***. To calculate the reinforcement strain based on the specified ***C*** value, a steel reinforcement strain diagram is obtained

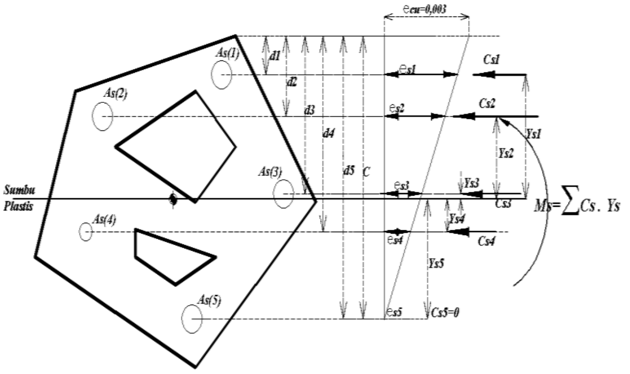


Figure 2. Reinforcement stress diagram of certain ***C*** values along with ***Cs & Ms***

In Figure 2. The steel strain is linearly related to the depth of the neutral axis ***C***. The reinforcement strain ***εs*** at a certain location is calculated based on the strain distribution as in the formula below:

$$\boldsymbol{\varepsilon si} = \left(\frac{C-di}{C}\right) \mathbf{0,003} \quad (11)$$

$$\boldsymbol{Cs} = \sum_{i=1}^{n} \boldsymbol{As(f's \; - \; 0,85\, f'c)} \quad (12)$$

εsi is the strain in the ***I-th*** steel reinforcement layer, at the depth of the I-th steel reinforcement layer from the end of the top pressed concrete fiber, ***C*** the height of the Neutral Axis from the end of the concrete pressed fiber. In Figure 3. The stress in compression reinforcement is the strain ***εs*** times the modulus of elasticity of the reinforcement ***Es*** is not allowed to exceed the value of ***fy***, because steel reinforcement behaves elastically only when the strain reaches yield ***εy***, so that when the steel compression strain εs' is equal to or greater than the yield strain ***εy*** then as the maximum limit of steel compressive stress and strain fs' is taken to be equal to the yield stress and strain ***fy***. The ***fs***' formula is expressed as

$$f'_s = E_s\, \varepsilon'_s\,, \; -fy \leq fsi \leq fy \quad (13)$$

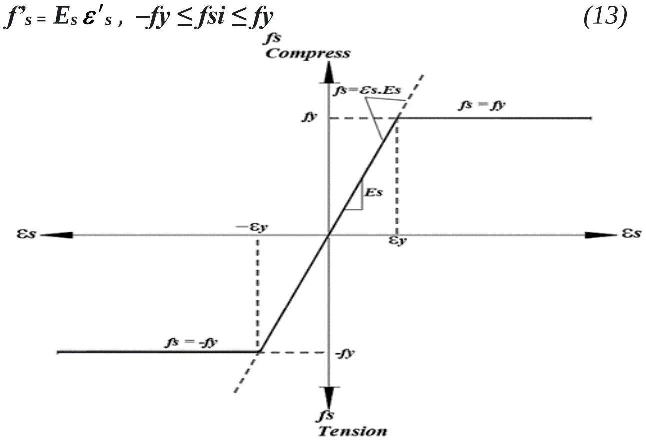


Figure 3. Image of the stress-strain relationship for reinforcing steel

If a is greater than the distance of a certain i-th layer of reinforcement from the pressed concrete fiber in, then the area of reinforcement in that layer has been included in the area of calculation of the area of pressed concrete ab which is used to calculate the concrete compressive force ***Cc***. For this reason, the ***f's*** value for the i-layer reinforcement is reduced by ***0.85 f'c***. Before calculating the ***Cs*** value, the positive ***f's*** value must be reduced by the value ***0.85.f'c***. Then the force in the compression reinforcement is expressed as:

So the axial force caused by the steel reinforcement is:

$$\boldsymbol{Cs} = \sum_{i=1}^{n} \boldsymbol{As\, fs} + \sum_{i=1}^{n} \boldsymbol{As(f's \; - \; 0,85\, f'c)} \quad (14)$$

Then the moment of resistance to the center of the column cross-section caused by the steel reinforcement either pulling or pushing against the plastic center of the cross-section is:

$$\boldsymbol{Ms} = \sum_{i=1}^{n} \boldsymbol{Cs\,.\; ys} + \sum_{i=1}^{n} \boldsymbol{As(f's \; - \; 0,85\, f'c).ys} \quad (15)$$

n is the number of layers of reinforcement, ***Cs*** is the axial force due to the reinforcement, ***As*** is the area of each reinforcement in the 1st layer, ***f's*** is the steel gap in the each layer which is $\boldsymbol{\varepsilon' s}$ multiplied by the modulus of elasticity ***Es, Ys*** The distance between each reinforcement to the ***X*** plastic axis

The unfactored axial force capacity of the column is calculated as follows:

$$\boldsymbol{Pn = Cc + Cs} \quad (16)$$

Then the unfactored bending moment of the column capacity is calculated as follows:

$$\boldsymbol{Mn = Mc + Ms} \quad (17)$$

After carrying out several iterations of the neutral axis ***C*** starting from a location as high as 3 times the cross-sectional height until it reaches the pressure fiber and is divided into 26 points, then the factored capacity for each value of ***ϕ Pn & ϕ Mn*** is calculated. These results just need to be plotted on a Cartesian graph to form a graph resembling a half clove of onion which is called the Reinforced Concrete Column Interaction Diagrams.

Compare results with the PCACOL output

To test the accuracy level of 26 calculation results with the output of the PCACOL program calculation results, the method is respectively ϕ Pn & ϕ Mn the results of manual calculations or a new program against the results of ϕ Pn & ϕ Mn PCACOL. To test the level of accuracy of the program results according to (Poli & Cirillo, 1993) a performance index test was carried out with NMSE (Normalized Mean Square Error) to measure how much coherence or harmony there is between the PCACOL output results and what has been assumed or the output results of the new program calculation, level its accuracy is proven by the distribution indication obtained from the use of the NMSE test. If the NMSE value is closer to zero, the calculated results are closer to harmony or accuracy between predictions and measurements. The formula used is:

$$\mathbf{NMSE} = \sum Si^2(1-ki)^2 \,/\, \sum Si\, ki \quad (18)$$

$$Si = Coi/Co \quad (19)$$

$$ki = Cpi/Coi \quad (20)$$

4. RESULTS AND DISCUSSION

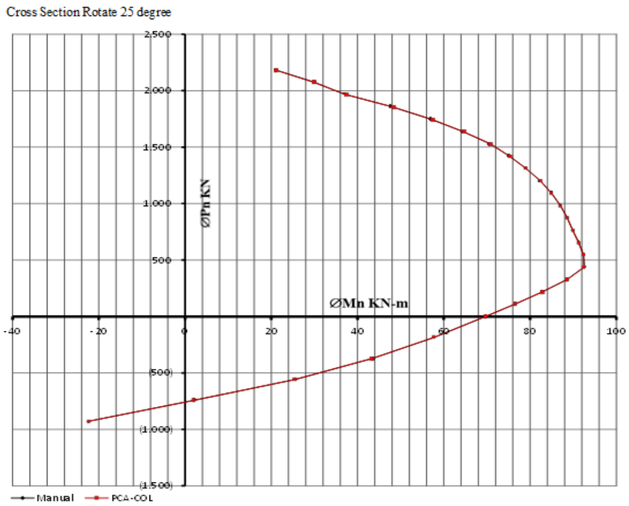
Table 4. Calculation of Pn & Mn for each iteration of neutral axis C and PCACOL output , angle 0^0

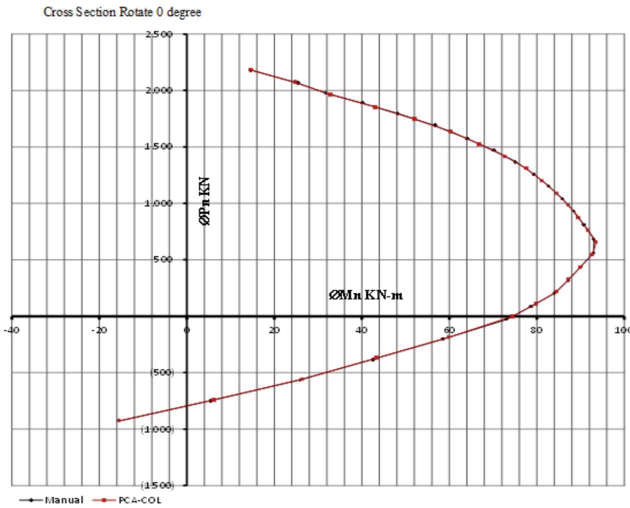
No	C (mm)	Ac (mm2)	Cy (mm)	Cc (N)	Mc (N-mm)	Cs (N)	Ms (N-mm)	Output Program Pn (N)	Output Program Mn (N-mm)	Output Pca-Col Pn (N)	Output Pca-Col Mn (N-mm)
1	767	55.000	0	1.309.000	36	872.786	14.650.030	2.181.786	14.650.065	2.181.830	14.650.000
2	401	55.000	0	1.309.000	36	755.212	25.468.248	2.064.212	25.468.284	2.072.730	24.700.000
3	332	54.536	1	1.297.947	1.553.504	679.049	30.192.539	1.976.996	31.746.043	1.963.640	32.850.000
4	305	52.504	6	1.249.594	7.430.377	639.146	32.667.676	1.888.740	40.098.052	1.854.550	42.990.000
5	285	50.000	11	1.190.008	13.539.385	605.176	34.774.748	1.795.185	48.314.133	1.745.460	52.160.000
6	267	47.018	18	1.119.026	19.689.112	570.346	36.935.215	1.689.372	56.624.326	1.636.370	60.430.000
7	248	43.412	25	1.033.198	25.795.814	539.376	38.277.150	1.572.574	64.072.964	1.527.280	66.740.000
8	232	40.784	30	970.651	29.399.933	499.742	40.735.625	1.470.393	70.135.558	1.418.190	72.680.000
9	219	38.131	35	907.519	32.115.784	462.451	43.048.723	1.369.970	75.164.507	1.309.100	77.560.000
10	208	35.083	41	834.973	34.534.899	421.701	44.865.374	1.256.674	79.400.273	1.200.000	81.270.000
11	198	32.611	46	776.150	35.964.569	377.718	46.778.840	1.153.868	82.743.408	1.090.910	84.420.000
12	188	29.851	52	710.461	37.026.482	331.391	48.794.288	1.041.852	85.820.770	981.820	87.210.000
13	178	27.133	58	645.774	37.530.125	281.719	50.955.237	927.492	88.485.362	872.730	89.490.000
14	168	24.355	65	579.645	37.491.738	228.204	53.349.736	807.848	90.841.474	763.640	91.590.000
15	158	21.926	71	521.846	36.964.613	164.028	56.141.653	685.875	93.106.266	654.550	93.560.000
16	146	19.412	78	462.008	35.868.834	102.817	57.104.339	564.825	92.973.173	545.460	92.600.000
17	131	16.700	85	397.457	33.943.194	41.557	56.079.461	439.015	90.022.655	436.370	89.960.000
18	117	14.923	91	355.156	32.147.481	- 37.201	54.920.908	317.955	87.068.389	327.270	87.300.000
19	113	13.545	94	322.383	30.412.397	- 118.820	53.655.953	203.562	84.068.350	218.180	84.510.000
20	106	12.109	98	288.205	28.280.872	- 200.286	50.430.630	87.918	78.711.502	109.090	79.720.000
21	95	10.777	102	256.492	26.045.653	- 285.029	47.014.838	- 28.536	73.060.490	-	74.450.000
22	87	7.626	110	181.494	19.883.126	- 383.992	38.625.580	- 202.498	58.508.706	- 185.610	60.000.000
23	55	5.142	116	122.389	14.249.765	- 501.900	28.340.186	- 379.512	42.589.951	- 371.220	43.380.000
24	45	3.399	122	80.888	9.891.308	- 644.720	15.881.790	- 563.832	25.773.098	- 556.830	26.410.000
25	29	1.423	131	33.860	4.444.624	- 780.941	1.029.659	- 747.081	5.474.283	- 742.440	6.070.000
26	-	-	-	-	-	- 928.046	- 15.572.093	- 928.046	- 15.572.093	- 928.050	- 15.570.000

Table 5. Calculation of Pn & Mn for each iteration of neutral axis C and PCACOL output , angle 25^0

No	C (mm)	Ac (mm2)	Cy (mm)	Cc (N)	Mc (N-mm)	Cs (N)	Ms (N-mm)	Output Program Pn (N)	Output Program Mn (N-mm)	Output Pca-Col Pn (N)	Output Pca-Col Mn (N-mm)
1	764	55.002	0	1.309.054	22	872.827	21.099.295	2.181.881	21.099.317	2.181.830	21.100.000
2	439	55.002	0	1.309.054	22	768.720	29.857.727	2.077.773	29.857.749	2.072.730	29.860.000
3	339	54.501	1	1.297.123	1.424.969	672.668	35.408.760	1.969.791	36.833.729	1.963.640	37.490.000
4	320	50.979	9	1.213.304	10.938.243	647.023	36.729.408	1.860.328	47.667.652	1.854.550	48.240.000
5	303	47.560	17	1.131.938	18.932.487	617.199	37.922.223	1.749.137	56.854.710	1.745.460	57.340.000
6	288	44.412	24	1.056.999	25.267.423	584.699	38.936.076	1.641.698	64.203.499	1.636.370	64.610.000
7	273	41.286	31	982.616	30.600.455	547.589	40.093.777	1.530.204	70.694.232	1.527.280	70.940.000
8	254	38.355	38	912.858	34.575.040	510.286	40.543.357	1.423.144	75.118.397	1.418.190	75.280.000
9	238	35.704	44	849.746	37.214.501	465.014	41.756.619	1.314.760	78.971.120	1.309.100	79.110.000
10	225	32.974	50	784.776	39.132.218	419.925	43.163.222	1.204.701	82.295.440	1.200.000	82.410.000
11	213	30.314	56	721.478	40.356.699	375.589	44.546.324	1.097.067	84.903.023	1.090.910	84.970.000
12	202	27.686	62	658.924	40.968.141	327.698	46.040.345	986.621	87.008.486	981.820	87.030.000
13	192	25.218	68	600.200	41.009.140	278.506	47.574.928	878.706	88.584.069	872.730	88.570.000
14	182	22.798	75	542.602	40.553.111	222.349	49.326.793	764.951	89.879.904	763.640	89.850.000
15	170	21.092	79	501.983	39.848.764	155.454	51.413.646	657.437	91.262.410	654.550	91.210.000
16	160	19.441	84	462.707	38.810.795	85.288	53.602.561	547.995	92.413.356	545.460	92.350.000
17	150	17.753	89	422.519	37.400.685	16.660	55.172.013	439.179	92.572.699	436.370	92.490.000
18	137	15.557	95	370.257	35.053.547	- 40.910	53.574.431	329.346	88.627.978	327.270	88.530.000
19	122	13.222	101	314.683	31.899.075	- 94.394	51.012.458	220.289	82.911.532	218.180	82.800.000
20	111	11.474	106	273.092	29.083.437	- 162.476	47.493.835	110.616	76.577.272	109.090	76.470.000
21	101	9.875	111	235.016	26.157.458	- 233.775	43.566.088	1.242	69.723.547	-	69.620.000
22	87	7.794	118	185.496	21.845.444	- 369.857	35.910.634	- 184.361	57.756.078	- 185.610	57.630.000
23	73	5.564	125	132.412	16.604.784	- 502.763	26.852.456	- 370.351	43.457.240	- 371.220	43.350.000
24	56	3.219	135	76.614	10.367.824	- 634.079	15.130.174	- 557.465	25.497.999	- 556.830	25.430.000
25	34	1.171	148	27.879	4.121.593	- 768.269	- 2.017.096	- 740.389	2.104.497	- 742.440	2.070.000
26	-	-	-	-	-	- 928.046	- 22.434.119	- 928.046	- 22.434.119	- 928.050	- 22.440.000

Gambar 5. Comparison graph of Output Program with Output PCACOL engle 0^0 and 25^0





4. CONCLUSIONS

Based on the test results of the accuracy level of 26 results of calculating the nominal capacity of axial force and nominal bending moment with the output of the calculation results of the PCACOL program using the NMSE (Normalized Mean Square Error) performance index test, the results obtained for measuring the nominal capacity of column axial forces against predictions from PCACOL obtained an index NMSE was 0.002144193 or 0.214%. Meanwhile, the NMSE test results for the nominal bending moment capacity of the column against predictions from PCACOL obtained an NMSE index of 0.000783626 or 0.078%. So it can be concluded that the performance index of alignment or accuracy between the manual calculation output results and what has been assumed, namely the PCACOL output results, has a high level of accuracy. Which also means that the closed polygon method with a clockwise exterior-boundary node numbering system and a clockwise interior boundary numbering system has been proven to be suitable for calculating interaction diagrams for reinforced concrete columns with irregular cross-sectional shapes accompanied by several holes in the middle.

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