The Birth of a Conjecture and the Death of an Incomplete Theorem

Deep Bhattacharjee

July 23, 2022

The birth of a conjecture and the death of an incomplete theorem

Deep Bhattacharjee[1]

When there lies a limitation in developing a theorem then there arises the fundamental generators of a conjecture which when proved develops the old incomplete theorem in the most beautiful, enriched, and pervasive ways ever humanity has seen.

Axioms – Theorems – Conjectures – Millennium Problems

Nature is best described by the laws of philosophy and when that combines with mathematics there arise the fundamental aspects describing nature in the purest form with explanations being symbolized through hypothesis, laws with axioms, and the theorems showing the symmetry of the systems. The conservation, the dissipative ones, the non–dissipative ones, the disorders through bifurcations make a way to the extremes of the 'patterns' being hidden in those disorders or other words – 'The existence of orders in disorders for every domain that physicists and mathematicians ever tried to construct by incorporating the numbers and symbols explain every ladder from the Planck to Multiverse[1-3].

In most cases – when the subject of explanation of mother nature goes beyond the limits and perceptions of humans but still they are trying in the best ways possible to interpret them, formulate them, compute them, and then make them chalked through pen and paper symbolizing the normality of every creation – some go on fully explainable while the most others are incomplete demands more mathematics to be discovered, more philosophies to be analyzed, more thinking to be developed – That incomplete theorem rises the conjectures with some the tough that couldn't at all be explained with others equally tough that some mathematician tried and succeeded to explain making the logics of the working of the universe in the eyes of this world in the most sustainable and magnificent forms ever been created giving birth to a new theorem[4].

Thus, an incompletion in making a theorem creates a conjecture which in turn develops the old incomplete theorem from which the conjecture has been assigned in the most profound ways ever possible[5,6].

[1]itsdeep@live.com

From the claymath page[7] there's given a complete list of the millennium prize problems along with their details,

[1] Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

[2] Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

[3] P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

[4] Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

[5] Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

[6] Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

[7] Birch and Swinnerton-Dyer Conjecture

Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles' proof of the Fermat Conjecture, factorization of numbers into primes, and cryptography, to name three.

The problem listed in number [6] that is Poincaré Conjecture has been solved and the details are[8]

In 1904, Henri Poincaré made a revolutionary conjecture in the mathematics of geometric topology asserting the similar principle holding for a 3D space when any compact boundariless 2D surface is homeomorphic to a 2-sphere for a continuous deformation of loops ending at a point[9-12]. After giving his initial form of conjecture – Poincare later extended it to any dimensions such that there exists a continuous deformation for any n–dimensional compact manifold is homotopy equivalent to n – sphere only if there is homoeomorphism to that n – sphere.

His initial conjecture when takes the dimension n = 3 then through dimensional increment the generalization to the above paragraph hold.

Denoting the conjecture in equivariant form from dimensions 1 to 5 and even greater than 5 there exists 5 points[13],

[1] $For\ dimension - 1 -$ The 1-dimensional manifold that is closed compact and simply connected, homeomorphism exists in the circle.

[2] $For\ dimension - 2 -$ This holds for the ordinary sphere.

[3] $For\ dimension - 3 -$ Perelman closed this by proving the conjecture.

[4] $For\ dimension - 4 -$ Michael Freedman proved the validation of the conjecture.

[5] $For\ dimension\ \geq 5 -$ Stephen Smale showed the validation of the conjecture and this dimension $\geq 5 -$ The generalized version of this conjecture has been shown in this paper taking C – isomorphism and inclusion maps for Kan – fibration and Kan – complex taking Whitehead group for vanishing Torsion in simple Homotopic equivalence for 'invariant in homotopy', 'invariant in topology' and CW – complex over homotopic equivalent for all – Differential manifold, Topological manifold, piecewise Linear manifold thereby controlling the entire relation by establishing it through Alexander trick.

Perelman took the finest steps of Ricci flow with surgery which got recognized in 2006 as a solution to the millennium prize problem. Describing surgery in the simplest steps one can induce a relation[14-18], any Riemannian 3-manifold being simply connected for metric g_R there exists a curve shortening flow over time occurring in $Ricci$ curvature where the maximum time for the evolution of the structure is always less than infinity. Throughout this temporal evolution for a particular initiation time provided the final time is always less than infinity we find two relations,

[1] Any region with negative curvature expands

[2] Any region with positive curvature contracts making metric g_R converge to that via the associated Ricci flow such that this will shrink – develops singularity and splits apart called surgery or topological surgery.

The other lists of conjectures are segregated by exampleproblems[19] being classified into a categorical structures as,

1. Proved (now theorems)

A. Adams conjecture
B. Bieberbach conjecture (now known as De Branges' theorem)
C. Blattner's conjecture (now often known as the Blattner formula)
D. Conway-Norton conjecture (now known as Monstrous moonshine)
E. Dinitz conjecture
F. Epsilon conjecture (an intermediate on the way to Fermat's last theorem)
G. Fermat's last theorem
H. Gradient conjecture
I. Heawood conjecture
J. Kummer's conjecture on cubic Gauss sums
K. Mahler-Manin conjecture
L. Manin-Mumford conjecture (now Raynaud's theorem)
M. Mordell conjecture (now known as Faltings' theorem)
N. Mumford conjecture (now Haboush's theorem)
O. Oppenheim conjecture
P. Ramanujan conjecture on the cusp form Δ
Q. Segal's Burnside ring conjecture
R. Serre's conjecture (now known as the Quillen-Suslin theorem)
S. Smith conjecture
T. Star height problem
U. Sullivan conjecture
V. Taniyama-Shimura conjecture
W. Wagner's conjecture (now known as the Robertson–Seymour theorem)
X. Weil conjectures

2. Disproved

A. Euler's conjecture
B. Ganea conjecture
C. Generalized Smith conjecture
D. Hauptvermutung
E. Intersection graph conjecture
F. Kouchnirenko's conjecture
G. Mertens conjecture
H. Ragsdale conjecture
I. Tait's conjecture
J. Von Neumann conjecture
K. Weyl-Berry conjecture

3. Recent work

A. Bloch-Kato conjecture
B. Catalan's conjecture
C. Erdős-Strauss conjecture
D. Hilbert-Smith conjecture
E. Kepler conjecture
F. Milnor conjecture
G. Nagata conjecture on automorphisms
H. Poincaré conjecture

I. Strong perfect graph conjecture
J. Thurston's conjecture

4. Open problems

A. abc conjecture
B. Andrews-Curtis conjecture
C. Agoh-Giuga conjecture
D. Artin conjectures
E. Atiyah conjecture
F. Bateman-Horn conjecture
G. Baum-Connes conjecture
H. Beal's conjecture
I. Beilinson conjecture
J. Berry-Tabor conjecture
K. Birch and Swinnerton-Dyer conjecture
L. Birch-Tate conjecture
M. Bloch-Beilinson conjectures
N. Borel conjecture
O. Bost conjecture
P. Collatz conjecture
Q. Cramér's conjecture
R. Deligne conjecture disambiguation
S. Eilenberg-Ganea conjecture
T. Elliott-Halberstam conjecture
U. Erdős-Gyárfás conjecture
V. Farrell-Jones conjecture
W. Fibonacci primes conjecture
X. Frankl conjecture
Y. Gilbreath conjecture
Z. Goldbach's conjecture
AA. Goldbach's weak conjecture
BB. Hadamard conjecture
CC. Hodge conjecture
DD. Homological conjectures in commutative algebra
EE. Jacobian conjecture
FF. Lawson's conjecture
GG. Lenstra-Pomerance-Wagstaff conjecture
HH. Lichtenbaum conjecture
II. List coloring conjecture
JJ. Littlewood conjecture
KK. Marshall Hall's conjecture
LL. Mazur's conjectures
MM. Monodromy conjecture
NN. New Mersenne conjecture
OO. Novikov conjecture
PP. Petersen coloring conjecture
QQ. Pierce-Birkhoff conjecture
RR. Pillai's conjecture
SS. De Polignac's conjecture
TT. Quillen-Lichtenbaum conjecture
UU. Reconstruction conjecture
VV. *Riemann Hypotheses*: see also Weil conjectures, above
    a. Riemann hypothesis

i. Generalized Riemann hypothesis
ii. Grand Riemann hypothesis
b. Density hypothesis
c. Lindelöf hypothesis
d. Hilbert-Pólya conjecture on the Riemann hypothesis
WW. Sato-Tate conjecture
XX. Schanuel's conjecture
YY. Schinzel's hypothesis H
ZZ. Scholz conjecture
AAA. Second Hardy-Littlewood conjecture
BBB. Selfridge's conjecture
CCC. Serre's multiplicity conjectures
DDD. Tate conjecture
EEE. Twin prime conjecture
FFF. Vandiver's conjecture
GGG. Weight-monodromy conjecture

References

[1] Bhattacharjee, D. (2022i). Establishing equivalence among hypercomplex structures via Kodaira embedding theorem for non-singular quintic 3-fold having positively closed (1,1)-form Kähler potential $i2^{-1}\partial\partial^*\rho$. *Research Square*. https://doi.org/10.21203/rs.3.rs-1635957/v1

[2] Bhattacharjee, D., Amani, D., Behera, A. K., Sadhu, R., & Das, S. (2022c). Young Sheldon's Rough Book on Strings – Decomplexifying Stuffs. *Authorea Preprint*. https://doi.org/10.22541/au.165057530.08903120/v1

[3] Bhattacharjee, D. (2021c). The Gateway to Parallel Universe & Connected Physics. *Preprints*. https://doi.org/10.20944/preprints202104.0350.v1

[4] Bhattacharjee, D. (2022a). A Coherent Approach Towards Quantum Gravity. *TechRxiv*. https://doi.org/10.36227/techrxiv.19785880.v1

[5] *[The Road to Reality] [By: Penrose, Roger] [February, 2006]*. (2004). Vintage Books.

[6] G. (2022). *Elegant Universe (05) by Greene, Brian [Paperback (2005)]*. Vintage s, Paperback(2005).

[7] *Millennium Problems | Clay Mathematics Institute*. (2022, July 22). Claymath. Retrieved July 23, 2022, from https://www.claymath.org/millennium-problems

[8] Bhattacharjee, D. (2022x). Generalized Poincaré Conjecture via Alexander trick over C-isomorphism extension to h-cobordism on inclusion maps with associated Kan-complex. *Research Square*. https://doi.org/10.21203/rs.3.rs-1830184/v1

[9] O'Shea, D. (2007). The Poincaré Conjecture: In Search of the Shape of the Universe (First Edition). Walker Books.

[10] Ricci Flow and the Poincare Conjecture (Clay Mathematics Monographs). (n.d.). Clay Mathematics Monographs.

[11] Perelman, G. (2003b). Ricci flow with surgery on three-manifolds. Differential Geometry (Math.DG). https://doi.org/10.48550/arXiv.math/0303109

[12] Perelman, G. (2002). The entropy formula for the Ricci flow and its geometric applications. arXiv:Math/0211159 [Math.DG]. https://doi.org/10.48550/arXiv.math/0211159

[13] Hosch, W. L. (n.d.). Poincaré conjecture | mathematics. Encyclopedia Britannica. Retrieved July 4, 2022, from https://www.britannica.com/science/Poincare-conjecture
[14] Perelman, G. (2003a). Finite extinction time for the solutions to the Ricci flow on certain three-manifolds. Differential Geometry (Math.DG). https://doi.org/10.48550/arXiv.math/0307245
[15] Dinkelbach, J. (n.d.-b). Equivariant Ricci Flow with Surgery. Google Books.
[16] Bhattacharjee, D. (2022m). Establishing equivalence among hypercomplex structures via Kodaira embedding theorem for non-singular quintic 3-fold having positively closed (1,1)-form Kähler potential $\boldsymbol{i}2-1\partial\partial^*\rho$. Research Square. https://doi.org/10.21203/rs.3.rs-1635957/v1
[17] Antoniou, S. (2019). Mathematical Modeling Through Topological Surgery and Applications (Springer Theses) (Softcover reprint of the original 1st ed. 2018 ed.). Springer.
[18] Bhattacharjee, D. (2022c). An outlined tour of geometry and topology as perceived through physics and mathematics emphasizing geometrization, elliptization, uniformization, and projectivization for Thruston's 8-geometries covering Riemann over Teichmuller spaces. TechRxiv. https://doi.org/10.36227/techrxiv.20134382.v1
[19] List of conjectures - Example Problems. (2022, July 22). In *exampleproblems*. http://exampleproblems.com/wiki/index.php/List_of_conjectures