Optimal Control of a Spatial Inverted Pendulum Using the Adjoint Method

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1 Introduction

The determination of critical parameters or control signals of a multibody system (MBS) is a common problem arising in the analysis and synthesis of dynamic systems. The indirect methods of optimal control constitute a powerful toolbox to address these complex non-linear problems. The adjoint method is one such approach, which has been employed in various applications, such as parameter identification [3] or sensitivity analysis of systems with flexible components [1]. This contribution presents how the adjoint method can be utilized to control complex electromechanical multibody system with closed-loop kinematic chain. Although the underlying dynamic problem is highly non-linear, we reported a satisfactory convergence of the optimization procedure.

2 Problem statement

Table 1: Model parameters

Parameter	Value
Links' 1–4 lengths	$l_i = 0.127$ m (5 inches)
Masses of bodies 1–4	$m_i = 0.065$ kg
Moment of inertia for bodies 1–4	$J_z = 9 \cdot 10^{-5}\,\mathrm{kg\,m^2}$
Pendulum's length	$l_5 = 0.3365$ m
Pendulum's mass	$m_5 = 0.125$ kg
Pendulum's moment of inertia	$J_x = 6.5 \cdot 10^{-6}\,\mathrm{kg\,m^2}$ $J_y = J_z = 1.8 \cdot 10^{-4}\,\mathrm{kg\,m^2}$
Transmission ratio	$k_g = 70$
Motor's moment of inertia	$J_m = 4.6 \cdot 10^{-7}\,\mathrm{kg\,m^2}$
Initial position of **P**	$\mathbf{P}^{(0)} = (0.127, 0.127)$ m

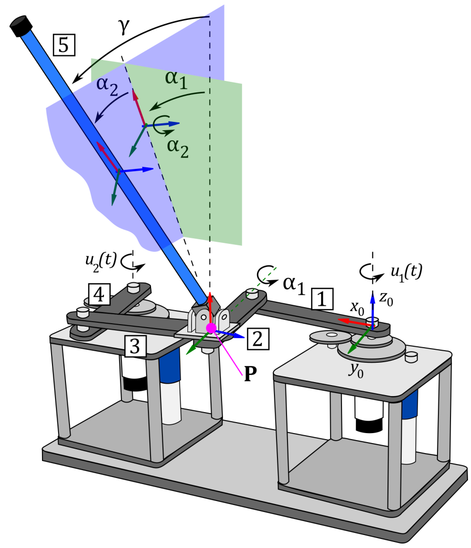

Figure 1: Motor-actuated five-bar linkage with inverted pendulum

The test model investigated in this paper is a spatial MBS composed of an inverted pendulum and a five-bar linkage. Its motion is modeled with a set of Hamilton's equations of motion in redundant coordinates [4]. The layout of the MBS is depicted in figure 1. The linkage is actuated by two DC motors that actuate bodies 1 and 4 via transmission modeled with constraint equation $\Phi^{\text{trans}} \equiv \varphi_{m_i} - k_g \cdot \varphi_i = 0$, $i = \{1, 4\}$. The motor torque is calculated with the following formula: $\tau_{m_i}(t) = g(V_i(t), \dot{\varphi}_{m_i}(t))$, where g is a known relation dependent on the voltage, motor actual velocity, and known motor parameters.

A physical pendulum is attached to the five-bar linkage at point **P** via a Hooke joint. The configuration of the pendulum can be conveniently described by means of joint coordinates $\{\alpha_1, \alpha_2\}$, which has been demonstrated in fig. 1. Angle γ denotes absolute value of the pendulum's inclination against global z axis.

At the initial time the pendulum is tilted about $\gamma \approx 14^\circ$ from global z axis ($\alpha_1 = \alpha_2 = 10^\circ$). The goal is to compute input voltage signals that stabilize the pendulum in the vertical position while avoiding the singular configurations of the five-bar. These criteria can be achieved by formulating the following performance measure:

$$J = \int_0^{t_f} \frac{1}{2}\left(\mathbf{P}-\mathbf{P}^{(0)}\right)^T\left(\mathbf{P}-\mathbf{P}^{(0)}\right)dt + \frac{1}{2}\gamma^2|_{t_f}, \tag{1}$$

where point $\mathbf{P}$ is depicted in figures 1 and at initial time $\mathbf{P} = \mathbf{P}^{(0)}$. The maneuver is supposed to end at final time $t_f = 0.5$ s, while the results of the forward dynamics problem are stored in computer memory with a constant step size of $\Delta t = 0.005$ s.

3 Simulation results

The continuous input voltage signals are discretized into a set of $k = 2 \cdot (\frac{t_f}{\Delta t} + 1)$ variables $\mathbf{b} \in \mathcal{R}^k$ of the non-linear programming problem. A cubic spline interpolation is employed when the integrator requests an intermediate input signal value. The starting guess is simply $\mathbf{b}_0 = \mathbf{0}$, which means no actuation from the motors. The SQP algorithm has been employed for the optimization.

The adjoint method consists of two main steps: MBS forward dynamics simulation and backward adjoint system integration [2]. The optimization took 14 iterations to converge, and the results can be seen in figure 2 presenting input voltage signals $u_1(t)$ and $u_2(t)$. Furthermore, figure 3 shows the dynamic response of the multibody system for the initial and final input signal vectors. The presented quantity is the total tilt of the pendulum from the vertical axis γ. One can notice that correct actuation properly stabilizes the pendulum.

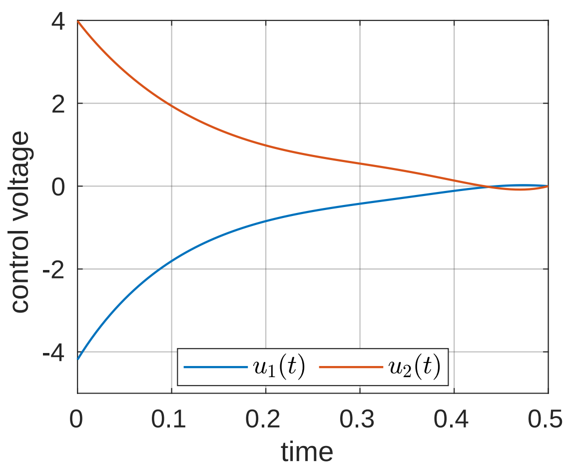

Figure 2: Computed input control signals that stabilize the pendulum in vertical position

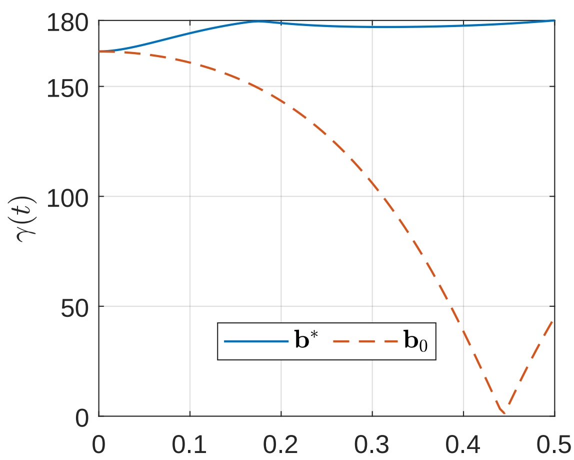

Figure 3: Angle γ for different vectors of input variables

Acknowledgments

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