The Generalized Dirichlet Problem for the Monge - Ampère Equation in Polydisc

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THE GENERALIZED DIRICHLET PROBLEM FOR THE MONGE - AMPÈRE EQUATION IN POLYDISC

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***Abstract* — This article examines the Dirichlet problem for the Monge-Ampère equation in the polydisc. It demonstrates that a solution exists for the Dirichlet problem when the boundary function is extended in a plurisubharmonic manner to a small neighborhood of the boundary of the domain. Based on the results of Walsh and Sadullaev, the continuity of the solution and its construction method are established. This study holds significant importance for both theoretical mathematical problems and applications.**

***Keywords* — subharmonic function, plurisubharmonic function, maximal function, the Dirichlet problem, pseudoconvex domain, strictly pseudoconvex domain**

I. INTRODUCTION

It is known that the classical Dirichlet problem involves finding a function that $u(z)$ is harmonic in D and continuous in ∂D, such that $u|_{\partial D} = \varphi$, where $D \subseteq \mathbb{R}^n$ is a bounded domain, and $\varphi(\xi) \in C(\partial D)$ is a given function. The Dirichlet problem arose in the process of solving physical problems and is considered one of the most widely applied problems in the plane. In the classical case, the Dirichlet problem for subharmonic functions has a solution for any regular domain. In particular, if the boundary of the domain is smooth, this problem has a unique solution. When considering the existence of a classical solution for the Dirichlet problem in an arbitrary bounded domain, the solution is generally based on the Dirichlet problem for a sphere, the maximum principle, and Perron's method for subharmonic functions (see [3]). For plurisubharmonic functions, this problem is solved under additional conditions.

II. STATEMENT OF THE PROBLEM

The foundation of pluripotential theory lies in plurisubharmonic (*Psh*) functions and their connection to the Monge-Ampère operator $(dd^c u)^n$. Here, as usual $d = \partial + \overline{\partial}$ and $d^c = \frac{\partial - \overline{\partial}}{4i}$. This theory has been extensively developed through the pioneering works of researchers such as E. Bedford, A. Taylor, J. Siciak, A. Sadullaev, and others (see [4],[5],[7]).

In the class of plurisubharmonic functions the Dirichlet problem is formulated as follows: for a domain $D \subset \mathbb{C}^n$ with boundary ∂D and a given function $\varphi \in C(\partial D)$, we need to find a function $u \in Psh(D)$ such that satisfies the boundary condition $u^*(\xi) = \varphi(\xi), \xi \in \partial D$ and is extremal among all such functions that satisfy this condition, i.e., for any other function v, $v \in Psh(D)$, $v|_{\partial D} \equiv \varphi(\xi)$, the inequality $v(z) \le u(z)$ holds in D.

In 1959, H.J.Bremermann (see [1]), using the Perron method, showed that if the domain is strictly pseudoconvex domain, then the problem has a solution. In 1968, J.B.Walsh (see [2]) demonstrated that this solution is continuous. In 1976, E. Bedford and B. A. Taylor (see [4]) proved that for a strictly pseudoconvex domain, the generalized Dirichlet problem for a function $\varphi \in C(\partial D)$ has a unique solution $u \in C(\overline{D})$, and this solution satisfies equation $(dd^c u)^n = 0$ in D. If the domain under consideration is not strictly pseudoconvex, then additional conditions must be imposed on the boundary function.

Now we consider the following Dirichlet problem in the polydisc $U \subset \mathbb{C}^n$

$$(dd^c u)^n = 0, \ u|_{\partial U} = \varphi(\xi),$$

where the function $\varphi(\xi)$ is a continuous function defined on ∂U. A.S. Sadullaev (see [5]) showed that this problem has a solution $u \in Psh(U)$satisfying the condition $\lim\limits_{z \to \xi, z \in U} u(z) = \varphi(\xi)$, $\xi \in \partial U$ and that this solution is unique and continuous.

III. THE DIRICHLET PROBLEM FOR THE MONGE - AMPÈRE EQUATION IN POLYDISC WITH ADDITIONAL CONDITIONS ON THE BOUNDARY FUNCTION

In this article, it is shown that the Dirichlet problem also has a solution when the boundary values of the function are given not on the entire boundary of the polydisc $U^2 \subset \mathbb{C}^2$, but only on its skeleton. First, we will examine this problem.

Theorem 1. *Let $U^2 \subset \mathbb{C}^2$ be a unit polydisc and let $T^2 \subset \overline{U^2}$ be its sceleton. Then for any arbitrary $\varphi(\xi) \in C(T^2)$ the Dirichlet problem $(dd^c u)^2 = 0$, $u|_{T^2} = \varphi(\xi)$ has a unique solution $u \in Psh(U^2) \cap C(U^2)$, $u|_{T^2} = \varphi(\xi)$.*

Proof: Since $\varphi(\xi) \in C(T^2)$, we construct the following function on U^2 using the Poisson integral

$$h(z_1, z_2) = \iint_{T^2} \varphi(\xi_1, \xi_2) P_1(\xi_1, z_1) P_2(\xi_2, z_2) d\xi_1 d\xi_2 \quad (1)$$

This function has the following properties:

1) the function $h(z_1, z_2)$ – 2-harmonic function, i.e at each fixed point $z_1 = z_1^0 \in U$ the function $h({z_1}^0, z_2)$ is harmonic with respect to z_2 and at each fixed point $z_2 = z_2^0 \in U$ the function $h(z_1, {z_2}^0)$ is harmonic with respect to z_1 .

2) $h(z_1, z_2) \in C(\overline{U^2})$. Initially, we define the function $\varphi(\xi)$ on the entire boundary ∂U^2. For this purpose, we first fix the point z_1^0 which satisfies $|z_1^0| = 1$ on the boundary ∂U^2

and take $\hat{\varphi}(z_1^0, z_2) = \int_{|\xi_2|=1} \varphi(z_1^0, \xi_2) P_2(\xi_2, z_2) d\xi_2$. This function is continuous and expresses a harmonic function with respect to z_2 and, in addition $\hat{\varphi}(z_1^0, z_2)|_{|z_2|=1} = \varphi(z_1^0, \xi_2)$. Similarly, if we fix the point z_2^0 which satisfies $|z_2^0| = 1$, the function $\hat{\varphi}(z_1, z_2^0) = \int_{|\xi_1|=1} \varphi(\xi_1, z_2^0) P_1(\xi_1, z_1) d\xi_1$ is continuous and expresses a harmonic function with respect to z_1, moreover $\hat{\varphi}(z_1, z_2^0)|_{|z_1|=1} = \varphi(\xi_1, z_2^0)$.

As a result the function

$$\hat{\varphi}(z_1, z_2) = \begin{cases} \int\limits_{|\xi_1|=1} \varphi(\xi_1, z_2) P_1(\xi_1, z_1) d\xi_1, & |z_2| = 1 \\ \int\limits_{|\xi_2|=1} \varphi(z_1, \xi_2) P_2(\xi_2, z_2) d\xi_2, & |z_1| = 1 \end{cases}$$

is defined on the entire boundary ∂U^2.

To show its contuinity on the boundary, we consider it at the arbitrary point $({z_1}^0, {z_2}^0)$ on the subset $\{|z_1| \le 1, |z_2| = 1\}$ of the boundary ∂U^2. For this, we consider the difference of the function as following:

$$|\hat{\varphi}(z_1, z_2) - \hat{\varphi}(z_1^0, z_2^0)| \le |\hat{\varphi}(z_1, z_2) - \hat{\varphi}(z_1^0, z_2)|$$

$$+|\hat{\varphi}(z_1^0, z_2) - \hat{\varphi}(z_1^0, z_2^0)|.$$

Using the continuity of the function $\varphi(\xi)$ on T^2, we see that, for any $\forall\varepsilon > 0$ there exists $\exists\delta_1 > 0$ such that

$$|\varphi(z_1, \xi_2) - \varphi(z_1^0, \xi_2)| < \varepsilon \qquad (2)$$

is true at the points $(z_1, \xi_2) \in T^2$, which satisfy $|\xi_2| = 1$ and $|z_1 - z_1^0| < \delta_1$, it follows that

$$|\hat{\varphi}(z_1, z_2) - \hat{\varphi}(z_1^0, z_2)| =$$

$$= \left| \int\limits_{|\xi_2|=1} \varphi(z_1, \xi_2) P_2(\xi_2, z_2) d\xi_2 - \int\limits_{|\xi_2|=1} \varphi(z_1^0, \xi_2) P_2(\xi_2, z_2) d\xi_2 \right|$$

$$\le \int\limits_{|\xi_2|=1} P_2(\xi_2, z_2) |\hat{\varphi}(z_1, z_2) - \hat{\varphi}(z_1^0, z_2)| d\xi_2 < 2\pi\varepsilon.$$

By the contuinity of the Poisson Kernel with respect to z_2, there exists $\exists\delta_2 > 0$ and we take the estimation

$$|\hat{\varphi}(z_1^0, z_2) - \hat{\varphi}(z_1^0, z_2^0)| =$$

$$= \left| \int\limits_{|\xi_2|=1} \varphi(z_1^0, \xi_2) P_2(\xi_2, z_2) d\xi_2 - \int\limits_{|\xi_2|=1} \varphi(z_1^0, \xi_2) P_2(\xi_2, z_2^0) d\xi_2 \right|$$

$$= \left| \int\limits_{|\xi_2|=1} \varphi(z_1^0, \xi_2) [P_2(\xi_2, z_2) - P_2(\xi_2, z_2^0)] d\xi_2 \right| < 2M\varepsilon$$

for $z_2 \in \{|z_2| \le 1\}$ which satisfies $|z_2 - z_2^0| < \delta_2$.

So, if we take $\delta = \min(\delta_1, \delta_2)$, at the arbitrary points $({z_1}^0, {z_2}^0) \in \{|z_1| \le 1, |z_2| \le 1\}$ which satisfy $|z_1 - z_1^0| < \delta_1$ and $|z_2 - z_2^0| < \delta_2$, we have

$$|\hat{\varphi}(z_1, z_2) - \hat{\varphi}(z_1^0, z_2^0)| < 2(\pi + M)\varepsilon. \qquad (3)$$

Now we show that the function $\hat{\varphi}(z_1, z_2)$ is the limit of the function $h(z_1, z_2)$ on the boundary. In particular, we show that the difference $|h(z_1, z_2) - \hat{\varphi}(z_1, \eta_2)|$ converges uniformly to 0 at $z_2 \to \eta_2$, $\eta_2 \in \{|z_2| = 1\}$. Based on (2), we evalute

$$|h(z_1, z_2) - \hat{\varphi}(z_1, \eta_2)|$$

$$= \left| \int\limits_{|\xi_1|=1} \int\limits_{|\xi_2|=1} \varphi(\xi_1, \xi_2) P_1(\xi_1, z_1) P_2(\xi_2, z_2) d\xi_1 d\xi_2 - \right.$$

$$\left. - \int\limits_{|\xi_1|=1} \varphi(\xi_1, \eta_2) P_1(\xi_1, z_1) d\xi_1 \right| =$$

$$= \left| \int\limits_{|\xi_1|=1} P_1(\xi_1, z_1) \left[\int\limits_{|\xi_2|=1} \varphi(\xi_1, \xi_2) P_2(\xi_2, z_2) d\xi_2 - \varphi(\xi_1, \eta_2) \right] d\xi_1 \right|$$

and we rewrite the integral inside as follows:

$$\left| \int\limits_{|\xi_2|=1} \varphi(\xi_1, \xi_2) P_2(\xi_2, z_2) d\xi_2 - \varphi(\xi_1, \eta_2) \right| =$$

$$= \left| \int\limits_{|\xi_2|=1} \varphi(\xi_1, \xi_2) P_2(\xi_2, z_2) d\xi_2 - \int\limits_{|\xi_2|=1} \varphi(\xi_1, \eta_2) P_2(\xi_2, z_2) d\xi_2 \right| =$$

$$= \left| \int\limits_{|\xi_2|=1} P_2(\xi_2, z_2) [\varphi(\xi_1, \xi_2) - \varphi(\xi_1, \eta_2)] d\xi_2 \right|.$$

Because of the continuity of the function $\varphi(\xi)$ on T^2, for $\forall\varepsilon > 0$ there exists $\exists\delta > 0$ such that $|\xi_2 - \eta_2| < \delta$ we have

$$|\varphi(\xi_1, \xi_2) - \varphi(\xi_1, \eta_2)| < \varepsilon \qquad (4)$$

at the points (ξ_1, ξ_2) which satisfy $|\xi_1| = 1$ and $|\xi_2 - \eta_2| < \delta$. Based on the property of Poisson Kernel $\int_{|\xi_2|=1} P_2(\xi_2, z_2) d\xi_2 = 1$ and for $\xi_2 \ne \eta_2$, $z_2 \in \{|z_2 < 1|\}$ we have $\lim\limits_{z_2 \to \eta_2} P_2(z_2, \xi_2) = 0$. From this follows that, for above ε and the points (ξ_1, ξ_2) such $|\xi_2 - \eta_2| > \delta$, there exists $\exists\delta' > 0$ such that, the inequality

$$|P_2(\xi_2, z_2)| < \varepsilon \qquad (5)$$

holds if $|z_2 - \eta_2| < \delta'$.

Now we denote the arc of the circle $\{|\xi_2| = 1\}$ that satisfies $|\xi_2 - \eta_2| < \delta$ with γ_1 and the arc that satisfies $|\xi_2 - \eta_2| > \delta$ with γ_2. So divide the integral

$\int_{|\xi_2|=1} P_2(\xi_2, z_2)[\varphi(\xi_1, \xi_2) - \varphi(\xi_1, \eta_2)]d\xi_2$ into integrals along arcs γ_1 and γ_2. According to (4), for the first arc $\forall \xi_1 \in \{|z_1| = 1\}$, $\forall z_2 \in \{|z_2| < 1\}$ we have

$$\left| \int_{\gamma_1} P_2(\xi_2, z_2)[\varphi(\xi_1, \xi_2) - \varphi(\xi_1, \eta_2)]d\xi_2 \right| < \varepsilon$$

and based on (5) for $\forall z_2 \in \{|z_2| < 1\}$ and $\forall \xi_1 \in \{|z_1| = 1\}$ which satisfy $|z_2 - \eta_2| < \delta'$, the following estimation

$$\left| \int_{\gamma_2} P_2(\xi_2, z_2)[\varphi(\xi_1, \xi_2) - \varphi(\xi_1, \eta_2)]d\xi_2 \right| < 2M \int_{\gamma_2} P_2(\xi_2, z_2)\, d\xi_2 \le 4M\pi\varepsilon$$

is valid, here $M = \max_{\xi \in T^2} |\varphi(\xi)|$.

If we take above inequality into account, as a result we get that the inequality

$$|h(z_1, z_2) - \hat{\varphi}(z_1, \eta_2)| < (1 + 4\pi M)\varepsilon$$

holds uniformly in $\{|z_1| \le 1\}$. This means that $\lim_{z_2 \to \eta_2} h(z_1, z_2) = \tilde{\varphi}(z_1, \eta_2)$. Like that $\lim_{z_1 \to \eta_1} h(z_1, z_2) = \hat{\varphi}(\eta_1, z_2)$ can be showed. So, the function $h(z_1, z_2)$ is continuous in $\overline{U^2}$.

3) Now we show the uniqueness of the function $h(z_1, z_2)$. We assume conversely, i.e let the functions $u_1(z)$ and $u_2(z)$ be harmonic functions defined by Poisson's integral (1). Then we consider $\vartheta(z) = u_1(z) - u_2(z)$. This function is also harmonic in U^2 and continuous in $\overline{U^2}$, and $\vartheta(z)|_{\partial U^2} = 0$. According to the maximum principle, this function reaches its maximum value in U^2, then $\vartheta = const$, due to its continuity $\vartheta = const$ is true also in $\overline{U^2}$. However, since $\vartheta \equiv 0$ in T^2, $\vartheta \equiv 0$ in $\overline{U^2}$ is followed. If the function reaches its maximum value in T^2, then again we have $\vartheta \equiv 0$.

Now we consider the following Dirichlet problem:

$$(dd^c u)^2 = 0,\ u|_{\partial U^2} \equiv \hat{\varphi}(\xi), \quad \hat{\varphi}(\xi) \in C(\partial U^2)$$

As usual, we search for the solution to this problem using Perron's method. We consider the class of the functions

$$\mathcal{U}(\hat{\varphi}, U^2) = \left\{ u : u \in Psh(U^2) \cap C(\overline{U^2}), u|_{\partial U^2} \le \hat{\varphi} \right\}$$

and take $\omega(z) = \sup_{u \in \mathcal{U}} u(z)$. In this case $\omega^*(z)$ is a maximal function, i.e $(dd^c \omega^*)^2 = 0$.

Now we need to show $\omega^*|_{\partial U^2} = \hat{\varphi}$. Instead of the above class we consider the class of subharmonic functions

$$\mathcal{U}_1(\hat{\varphi}, U^2) = \left\{ u : u \in sh(U^2) \cap C(\overline{U^2}), u|_{\partial U^2} \le \hat{\varphi} \right\}.$$

In this case, such a harmonic function $F(z)$ is found, that $\Delta F = 0$, $F|_{\partial U^2} = \hat{\varphi}$ holds. Function $F(z)$ satisfies $F(z) \ge \omega(z)$, from which yields

$$\overline{\lim_{z \to \xi}}\, \omega(z) \le \hat{\varphi}(\xi). \qquad (6)$$

On the other hand, we fix the arbitrary boundary point $\xi^0 = (\xi_1^0, \xi_2^0) \in \partial U^2$ and without loss of generality we consider $\xi_2^0 = 1$. In this case, function $h(z_1, 1)$ is harmonic in $|z_1| < 1$ and continuous in $|z_1| \le 1$. Using this function, we take following function:

$$v(z) = h(z_1, 1) + c\mathrm{Re}(z_2 - 1) - \varepsilon,\ c > 0, \varepsilon > 0.$$

This function belongs to the class $\mathcal{U}$ at sufficiently large c. Moreover, $\lim_{z \to \xi^0, z \in U^2} v(z) = \hat{\varphi}(\xi^0) - \varepsilon$. Due to the arbitrariness of ε and ξ^0,

$$\underline{\lim}_{z \to \xi, z \in U^2} \omega(z) \ge \hat{\varphi}(\xi),\ \forall \xi \in \partial U^2 \qquad (7)$$

holds. So, from the relations (6) and (7) it is followed that $\lim_{z \to \xi, z \in U^2} \omega(z) = \hat{\varphi}(\xi), \forall \xi \in \partial U^2$ is true and $\lim_{z \to \xi, z \in U^2} \omega(z) = \varphi(\xi)$, $\forall \xi \in T^2$ holds. Since the function $\omega^*(z)$ is plurisubharmonic in U^2, it holds the continuity condition $\lim_{z \to \xi, z \in U^2} \omega^*(z) = \varphi(\xi)$, $\forall \xi \in T^2$ on the boundary. Now we show the uniqueness. Assume that, the function $v(z)$ is the another solution of the generalized Dirichlet problem $(dd^c v)^2 = 0$, $v|_{\partial U^2} = \omega^*|_{\partial U^2}$. Then $v(z)$ is maximal in U^2 and so $v(z) \ge \omega^*(z)$ is true in U^2. However, ω^* is also maximal in U^2, i.e $\omega^* \ge v$. It follows that, $\omega^*(z) \equiv v(z)$. The theorem is proved.

Now we consider the Dirichlet problem for the Monge-Ampère equation on the unit polydisc $U \subset \mathbb{C}^n$:

$$(dd^c u)^n = 0, \qquad u|_{\partial U} \equiv \varphi(\xi), \xi \in \partial U,$$

where $\varphi(z) \in Psh(G) \cap C(\bar{G})$ and $G = O \cap U$, $O -$ sufficiently small neighbourhood of ∂U .

The main result of the article is the following theorem.

Theorem 2. *Let $U \subset \mathbb{C}^n$ be a unit polydisc and $G = O \cap U$, where $O -$ sufficiently small neighbourhood of ∂U and $\varphi(z) \in Psh(G) \cap C(\bar{G})$. Then the Dirichlet problem $(dd^c u)^n = 0$, $u|_{\partial U} = \varphi(\xi)$ has a unique solution $u \in Psh(U) \cap C(\overline{U})$.*

Proof. As usual, we search for the solution to this problem using the Perron's method. We consider the class of plurisubharmonic functions $\mathcal{U}(\varphi, U) = \{\vartheta \in Psh(U) \cap C(\overline{U}),\ \vartheta(z)|_{\partial U} \le \varphi(\xi)\}$ and define $\omega(z) = \sup\{\vartheta(z) : \vartheta \in \mathcal{U}(\varphi, U)\}$. Then ω^* regularization represents the maximal function in U, i.e. $(dd^c \omega^*)^n = 0$.

Now we show that function satisfies the boundary condition $\omega^*|_{\partial U} = \varphi$. First, in order to show $\overline{\lim}_{z\to\xi, z\in U} \omega^*(z) \le \varphi(\xi),\ \ \xi \in \partial U$, we compare $\omega^*(z)$ with the solution $F(z) \in h(U) \cap C(\overline{U})$ of Laplace's equation $\Delta F = 0$ in U, which satisfies $F|_{\partial U} = \varphi$. There is such a solution to the Dirichlet problem for a regular domain U. Since $F(z)$ is subharmonic in U and as the upper envelope of subharmonic functions v satisfying condition $v|_{\partial U} \le \varphi$, then it satisfies $F(z) \ge \omega^*(z)$. It follows that $\overline{\lim}_{z\to\xi, z\in U} \omega^*(z) \le \varphi(\xi),\ \ \xi \in \partial U$. On the other hand, since U is pseudoconvex, it can be covered by strictly pseudoconvex domains $\emptyset \ne G_1 \subset\subset G_2 \subset\subset \cdots,\ \ \cup G_j = U$. In particular, if we take function

$$\rho(z) = -\ln[(1 - |z_1|^2)(1 - |z_2|^2)\dots(1 - |z_n|^2)],$$

then domains $G_j = \left\{\rho(z) < \frac{1}{j}\right\}$ are strictly pseudoconvex domains. According to the condition, there exists a number $j_0 \in N$ such that for any $j > j_0$ it holds $\partial G_j \subset G$ and since $\varphi(z) \in Psh(G) \cap C(\overline{G})$ we take a sequence $\varphi_j(z) = \varphi|_{\partial G_j}$ which converges to $\varphi(\xi),\ \xi \in \partial U$. For the sequence of domains $G_j, j > j_0$ we consider the classes:

$$\mathcal{U}(\varphi_j, G_j) = \{u(z) \in Psh(G_j) \cap C(\overline{G_j}), u(z)|_{\partial G_j} \le \varphi_j\}$$

and put $\omega_j = \sup\{u(z) : u \in \mathcal{U}(\varphi_j, G_j)\}$. Then based on the fact, that the Dirichlet problem has a solution for strictly pceudoconvex domains, there exist solutions $\omega_j^* \in Psh(G_j) \cap C(\overline{G}_j)$ and $\lim_{z\to\xi\in\partial G_j} \omega_j^*(z) = \varphi_j$.

Additionally, $\left|\omega_j^*(z)\right| \le \max_{\xi\in\partial U} |\varphi(\xi)|$. The sequence $\{\omega_j{}^*\}$ is monotonically increasing and upper locally uniformly bounded. So there exists $\lim_{j\to\infty} \omega_j^* = \omega_0^*$ and $\omega_o^* \in Psh(U) \cap C(\overline{U})$, in addition to that $\lim_{z\to\xi\in\partial U} \omega_0^*(z) = \varphi(\xi)$. Now we choose an arbitrary boundary point $\overset{\circ}{\xi} = ('\overset{\circ}{\xi}, \overset{\circ}{\xi}_n) \in \partial U$ and without loss of generality, assume that $\overset{\circ}{\xi}_n = 1$. Define the function

$$v(z) = \omega_0^*('z, 1) + cRe(z_n - 1) + \varepsilon,\ c > 0,\ \varepsilon > 0\ .$$

This function belongs to the class $\mathcal{U}(\varphi, U)$ for sufficiently large c. Moreover, $\lim_{z\to\xi^0, z\in U} v(z) = \varphi(\overset{\circ}{\xi}) + \varepsilon$. Due to arbitrariness ε it follows that $\lim_{z\to\xi^0, z\in U} \omega^*(z) \ge \varphi(\overset{\circ}{\xi})$ and since $\overset{\circ}{\xi}$ arbitrary we have $\lim_{z\to\xi} \omega^*(z) = \varphi(\xi)$.

The continuity of ω^* follows from Walsh's result (see [2]). The uniqueness of the solution is demonstrated as in Theorem 1. The theorem is proved.

IV. CONCLUSION

In this article, the Dirichlet problem for the Monge-Ampère equation in the polydisc has been thoroughly examined. The existence of a solution was demonstrated under the condition that the boundary function is extended in a plurisubharmonic manner to a small neighborhood of the boundary of the domain. Using the Perron method and the results of Walsh and Sadullaev, the continuity and uniqueness of the solution were established.

This study contributes significantly to pluripotential theory by providing a robust framework for solving the generalized Dirichlet problem in pseudoconvex and strictly pseudoconvex domains. The results are not only theoretical but also provide tools for addressing practical problems in complex analysis and potential theory. Future work may focus on exploring more general domains, refining boundary conditions, or extending the method to higher-dimensional complex spaces.

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